CHECK YOUR GRASP PRINCIPLE OF MATHEMATICAL INDUCTION EXERCISE-I

1. The sum of n terms of $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + ...$ is-

(1) $\frac{n(n+1)(2n+1)}{6}$ (2) $\frac{n(n+1)(2n-1)}{6}$

(3) $\frac{1}{12}n(n+1)^2(n+2)$ (4) $\frac{1}{12}n^2(n+1)^2$

2. The greatest positive integer. which divides $(n + 16)(n + 17)(n + 18)(n + 19)$, for all $n \in N$, is-

(1) 2 (2) 4 (3) 24 (4) 120

3. Let $P(n) : n^2 + n$ is an odd integer. It is seen that truth of $P(n) \Rightarrow$ the truth of $P(n + 1)$. Therefore, $P(n)$ is true for all-

(1) $n > 1$ (2) n

(3) $n > 2$ (4) None of these

4. For every natural number n-

(1) $n > 2^n$ (2) $n < 2^n$ (3) $n \ge 2^n$ (4) $n \le 2^n$

5. If $n \in N$, then $x^{2n-1} + y^{2n-1}$ is divisible by-

(1) $x + y$ (2) $x - y$ (3) $x^2 + y^2$ (4) $x^2 + xy$

6. The inequality $n! > 2^{n-1}$ is true-

(1) for all $n > 1$ (2) for all $n > 2$

(3) for all $n \in N$ (4) None of these

7. $1.2^2 + 2.3^2 + 3.4^2 + ...$ upto n terms, is equal to-

(1) $\frac{1}{12}n(n+1)(n+2)(n+3)$

(2) $\frac{1}{12}n(n+1)(n+2)(n+5)$

(3) $\frac{1}{12}n(n+1)(n+2)(3n+5)$

(4) None of these

8. The sum of the cubes of three consecutive natural numbers is divisible by-

(1) 2 (2) 5 (3) 7 (4) 9

9. If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by-

(1) 113 (2) 123

(3) 133 (4) None of these

10. If $n \in N$, then $3^{4n+2} + 5^{2n+1}$ is a multiple of-

(1) 14 (2) 16 (3) 18 (4) 20

11. For each $n \in N$, $10^{2n+1} + 1$ is divisible by-

(1) 11 (2) 13

(3) 27 (4) None of these

12. The difference between an +ve integer and its cube is divisible by-

(1) 4 (2) 6

(3) 9 (4) None of these

13. If n is a natural number then $\left(\frac{n+1}{2}\right)^n \ge n!$ is true when-

(1) $n > 1$ (2) $n \ge 1$ (3) $n > 2$ (4) Never

14. For natural number n, $2^n (n - 1)! < n^n$, if-

(1) $n < 2$ (2) $n > 2$ (3) $n \ge 2$ (4) never

15. For every positive integer n, $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2n^3}{3} - \frac{n}{105}$ is-

(1) an integer

(2) a rational number

(3) a negative real number

(4) an odd integer

16. For positive integer n, $3^n < n!$ when-

(1) $n \ge 6$ (2) $n > 7$ (3) $n \ge 7$ (4) $n \le 7$

17. If $A = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$, then for any $n \in N$, A^n equals-

(1) $\begin{pmatrix} na & n \\ 0 & na \end{pmatrix}$ (2) $\begin{pmatrix} a^n & na^{n-1} \\ 0 & a^n \end{pmatrix}$

(3) $\begin{pmatrix} na & 1 \\ 0 & na \end{pmatrix}$ (4) $\begin{pmatrix} a^n & n \\ 0 & a^n \end{pmatrix}$

18. The sum of n terms of the series

$$\frac{\frac{1}{2}\cdot\frac{2}{2}}{1^3} + \frac{\frac{2}{2}\cdot\frac{3}{2}}{1^3+2^3} + \frac{\frac{3}{2}\cdot\frac{4}{2}}{1^3+2^3+3^3} + ...$$ is-

(1) $\frac{1}{n(n+1)}$ (2) $\frac{n}{n+1}$ (3) $\frac{n+1}{n}$ (4) $\frac{n+1}{n+2}$

19. For all $n \in N$, $7^{2n} - 48n - 1$ is divisible by-

(1) 25 (2) 26 (3) 1234 (4) 2304

20. The n^{th} term of the series

$4 + 14 + 30 + 52 + 80 + 114 + ...$ is-

(1) $5n - 1$ (2) $2n^2 + 2n$ (3) $3n^2 + n$ (4) $2n^2 + 2$

21. If $10^n + 3.4^{n+2} + \lambda$ is exactly divisible by 9 for all $n \in N$, then the least positive integral value of λ is-

(1) 5 (2) 3 (3) 7 (4) 1

22. The sum of n terms of the series

$1 + (1 + a) + (1 + a + a^2) + (1 + a + a^2 + a^3) + ...$, is-

(1) $\frac{n}{1-a} - \frac{a(1-a^n)}{(1-a)^2}$ (2) $\frac{n}{1-a} + \frac{a(1-a^n)}{(1-a)^2}$

(3) $\frac{n}{1-a} + \frac{a(1+a^n)}{(1-a)^2}$ (4) $-\frac{n}{1-a} + \frac{a(1-a^n)}{(1-a)^2}$

23. For all $n \in N$, n^4 is less than-

(1) 10^n (2) 4^n

(3) 10^{10} (4) None of these

24. For all $n \in N$, Σn

(1) $< \dfrac{(2n+1)^2}{8}$ (2) $> \dfrac{(2n+1)^2}{8}$

(3) $= \dfrac{(2n+1)^2}{8}$ (4) None of these

25. For all $n \in N$, $\cos\theta \cos2\theta \cos4\theta \ldots\ldots \cos2^{n-1}\theta$ equals to-

(1) $\dfrac{\sin 2^n\theta}{2^n \sin\theta}$ (2) $\dfrac{\sin 2^n\theta}{\sin\theta}$

(3) $\dfrac{\cos 2^n\theta}{2^n \cos 2\theta}$ (4) $\dfrac{\cos 2^n\theta}{2^n \sin\theta}$

26. For all positive integral values of n, $3^{2n} - 2n + 1$ is divisible by-

(1) 2 (2) 4 (3) 8 (4) 12

27. $\dfrac{1^2}{1} + \dfrac{1^2+2^2}{1+2} + \dfrac{1^2+2^2+3^2}{1+2+3} + \ldots$ upto n terms is-

(1) $\dfrac{1}{3}(2n + 1)$ (2) $\dfrac{1}{3}n^2$

(3) $\dfrac{1}{3}(n + 2)$ (4) $\dfrac{1}{3}n(n + 2)$

28. The smallest positive integer for which the statement $3^{n+1} < 4^n$ holds is-

(1) 1 (2) 2 (3) 3 (4) 4

29. Sum of n terms of the series

$\dfrac{1}{1} + \dfrac{1}{1+2} + \dfrac{1}{1+2+3} + \ldots\ldots$ is-

(1) $\dfrac{n}{n+1}$ (2) $\dfrac{2}{n(n+1)}$ (3) $\dfrac{2n}{n+1}$ (4) $\dfrac{2(n+1)}{n+2}$

30. For every natural number n, $n(n + 3)$ is always-

(1) multiple of 4 (2) multiple of 5

(3) even (4) odd

31. $\dfrac{1}{1.3} + \dfrac{1}{3.5} + \dfrac{1}{5.7} + \ldots\ldots$ upto n terms is-

(1) $\dfrac{1}{2n+1}$ (2) $\dfrac{n}{2n+1}$ (3) $\dfrac{1}{2n-1}$ (4) $\dfrac{2n}{3(n+1)}$

32. For positive integer n, $10^{n-2} > 81n$ when-

(1) $n < 5$ (2) $n > 5$ (3) $n \geq 5$ (4) $n > 6$

33. If P is a prime number then $n^p - n$ is divisible by p when n is a

(1) natural number greater than 1

(2) odd number

(3) even number

(4) None of these

34. $1 + 3 + 6 + 10 + \ldots\ldots$ upto n terms is equal to-

(1) $\dfrac{1}{3}n(n + 1)(n + 2)$ (2) $\dfrac{1}{6}n(n + 1)(n + 2)$

(3) $\dfrac{1}{12}n(n + 2)(n + 3)$ (4) $\dfrac{1}{12}n(n + 1)(n + 2)$

35. A student was asked to prove a statement by induction. He proved

(i) P(5) is true and

(ii) Truth of $P(n) \Rightarrow$ truth of $p(n + 1)$, $n \in N$

On the basis of this, he could conclude that P(n) is true for

(1) no $n \in N$ (2) all $n \in N$

(3) all $n \geq 5$ (4) None of these

36. The sum of the series

$\dfrac{3}{1^2} + \dfrac{5}{1^2+2^2} + \dfrac{7}{1^2+2^2+3^2} + \ldots\ldots$ upto n terms

(1) $\dfrac{2n}{n+1}$ (2) $\dfrac{3n}{n+1}$ (3) $\dfrac{3n}{2(n+1)}$ (4) $\dfrac{6n}{n+1}$

37. $\dfrac{1}{2} + \dfrac{3}{4} + \dfrac{7}{8} + \dfrac{15}{16} + \ldots$ upto n terms equal to-

(1) $n + \dfrac{1}{2^n}$ (2) $2n + \dfrac{1}{2^n}$

(3) $n - 1 + \dfrac{1}{2^n}$ (4) $n + 1 + \dfrac{1}{2^n}$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	3	4	2	1	2	3	4	3	1	1	2	2	2	1
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	2	2	4	3	1	1	1	1	1	1	4	4	3	3
Que.	31	32	33	34	35	36	37								
Ans.	2	3	1	2	3	4	3								

PREVIOUS YEAR QUESTIONS — PRINCIPLE OF MATHEMATICAL INDUCTION — EXERCISE-II

1. Let $S(k) = 1 + 3 + 5 + \ldots\ldots + (2k - 1) = 3 + k^2$, then which of the following is true ? [AIEEE-2004]

(1) S(1) is true

(2) $S(k) \Rightarrow S(k+1)$

(3) $S(k) \not\Rightarrow S(k+1)$

(4) Principle of mathematical Induction can be used to prove that formula

2. The sum of first n terms of the given series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$ is $\frac{n(n+1)^2}{2}$,

when n is even. When n is odd, then sum will be- [AIEEE-2004]

(1) $\frac{n(n+1)^2}{2}$ (2) $\frac{1}{2}n^2(n+1)$

(3) $n(n+1)^2$ (4) None of these

3. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, (by the principal of mathematical induction) [AIEEE-2005]

(1) $A^n = nA + (n-1)I$ (2) $A^n = 2^{n-1}A + (n+1)I$

(3) $A^n = nA - (n-1)I$ (4) $A^n = 2^{n-1}A - (n-1)I$

4. **Statement –1** : For every natural number $n \geq 2$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$$

Statement –2 : For every natural number $n \geq 2$,

$\sqrt{n(n+1)} < n+1$. [AIEEE-2008]

(1) Statement –1 is false, Statement –2 is true

(2) Statement–1 is true, Statement–2 is false

(3) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1

(4) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1

5. **Statement - 1:** For each natural number n, $(n+1)^7 - n^7 - 1$ is divisible by 7.

Statement - 2: For each natural number n, $n^7 - n$ is divisible by 7. [AIEEE-2011]

(1) Statement-1 is false, statement-2 is true.

(2) Statement-1 is true, statement-2 is true; Statement-2 is correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is true; Statement-2 is not a correct explanation for statement-1.

(4) Statement-1 is true, statement-2 is false.

ANSWER KEY

Que.	1	2	3	4	5
Ans.	2	2	3	3	2